# Optical measurement techniques with telecentric lenses 

## 1. Introduction

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This course on optical 2d-measurement technique has been created during the development of the telecentric lenses from
Schneider-Kreuznach. First of all we had the idea to explain to the "non optics expert" the differencies between normal lenses and telecentric optical systems by introducing the laws of perspective (Chapter 2).
But in the course of writing it became very soon evident that we had to deal with more then only explaining the laws of telecentric perspective. This because of various reasons:

First of all there was a need to explain the relationship between the pure mathematical central projection (which is used in optical measurement technique in form of the "linear camera model" - originally developed in photogrammetry) and the ideal optical imaging by Gaussian Optics.
The goal has been to give an answer to the frequently arising question: "what shall be considered as center of projection in optics - the pupils or the principal points?". To this end it has been necessary to introduce the principles of Gaussian Optics as well as the principles of ray limitations in Gaussian approximation.
In the framework of such a "short course" this has been only possible in a very condensed form. We encourage the interested reader to go into more detail with the help of cited literature (chapter 7).

The second reason for expanding the goal of this course has been to demonstrate the consequences of the ray bundles with finite aperture - in contrast to mathematical central projection with-one dimensional projection rays.
The first of these consequences is concerned with the region of "sharp imaging", i.e. geometric optical depth of field and depth of focus.
The second consequence of finite ray bundles is revealed if one considers the wave nature of light. Even with perfect image formation (without any aberrations) object points will be imaged to "points" with a certain extension. The extension of these image discs is depending on the wavelength of light and on the finite aperture of the ray bundles. This leads to certain modifications for the geometric optical depth of field, which will be treated in a short overview. (this is all found in chapter 3)

After these preliminary discussions we were able to enter into a more detailed treatment on the specific properties of telecentric perspective and is advantages for optical measurement technique. Here we have to distinguish between object side telecentric systems and the object and image side telecentric systems. The latter will be called bilateral telecentric lenses in the remainder of this course.

They possess similar advantages for optical measurement technique not only on the object side, but in addition on the image side. This will be discussed in detail particularly because all the telecentric lenses from Schneider Kreuznach are bilateral (chapter 4).

In chapter 5 we try to give an overview on all possible error sources (to a first approximation) which may arise by the information transfer in optical 2d-measurement techniques.
Here again the advantages of bilateral telecentric systems will be very evident, because they avoid "per se" a large number of error sources.

The last chapter 6 gives an introduction and some explanations to the technical datasheets of the telecentric lenses from Schneider Kreuznach which may be found on this WEB-site. Based on this, the present shortcourse has the following structure:

## Optical measurement technique with telecentric lenses

1. Introduction (this page)
2. What means telecentric? - an introduction into the laws of perspective
3. Gaussian Optics and its relation to mathematical central projection
3.1 The mathematical model of central projection
3.2 The ideal optical imaging (Gaussian Optics) and mathematical central projection
3.3 The limitations of ray bundles
3.4 The connection with the linear camera model
3.5 Consequences of finite ray bundles I: geometric optical depth of field and depth of focus
3.6 Consequences of finite ray bundles II:
the wave nature of light - diffraction effects
3.6.1 diffraction limited (perfect) imaging 3.6.2diffraction and depth of field (or focus)
4. Central projection from infinity - telecentric perspective
4.1 object side telecentric systems
4.2 bilateral (object and image side) telecentric systems
5. Error budget for 2d-optical measurement techniques
5.1 Centre of projection infinite distance (entocentric perspective)
5.2 Centre of projection at infinity (telecentric perspective)
6. Introduction to the bilateral telecentric lenses from Schneider Kreuznach
7. Literature

## Finally we have the following hint and invitation:

This short course will enable you - as our customer - to use our products with best efficiency. Nevertheless we are the opinion that "nobody is perfect" and that here are always possibilities to improve the form, design and content as well as the level of this course.
So, if you have some proposals, wishes for additional matters or constructive points of critique, please don't hesitate to contact us.

## Your Schneider-Kreuznach team

## Optical measurement techniques with telecentric lenses

## 2. What are telecentric lenses? - an introduction to the laws of perspective

Nearly everybody has had the experience of taking extreme tele- and wide angle photos. By taking pictures with extreme telephoto-lenses (long focal length lenses) the impression of spatial depth in the image seems to be very flat, e.g. on a race course, where the cars have in reality some distance, they seem to stick to each other. Whereas with extreme wide angle lenses (short focal lengths) the impression of spatial depth seems to be exaggerated. As an example we look at the two photos of a chess board.


Fig. 2-1: Wideangle- and Teleperspective
Both photos have been taken with a 35 mm format camera ( $24 \times 36 \mathrm{~mm}$ ), the left one with a moderate wide angle lens (focal length 35 mm ) and the right one with a tele-lens of 200 mm focal length. In both pictures the foreground (e.g. the front edge of the chess board) has about the same size. To realize this, one has to approach the object very closely with the wide angle lens (approx. 35 cm ). With the tele-lens however the distance has to be very large (approx. 2 m ). One may clearly observe, that with the wide angle lens the chess figures, which were arranged in equally spaced rows, seem to become rapidly smaller, thus giving the impression of large spatial depth. In contrast, with the tele-lens picture the checker figures seem to reduce very little, thus leading to the impression that the spatial depth is much lower. What are the reasons for this different impressions of the same object?

In order to give a clear understanding for these reasons, we look away from all (more or less complicated) laws of optical imaging by lenses and look at the particularly simple case of a pinhole camera ("camera obscura"). In the past the old painters - e.g. Dürer - used this resource to study the laws of perspective. Imaging by the pinhole-camera is a simple model for imaging by lens systems. In chapter 3 we will explain how this model is connected to real imaging with lenses.

As the name indicates, a pinhole-camera consists of a square box with a very small hole at the front side and on the rear side we have a focussing screen or a light sensitive plate. This rear side plane we will call the image plane. The size of the hole in the front side we imagine - at this instance very naively - to be so small, that only one single ray of light may pass through. The distance between the front and rear side of the camera we call the image width $\mathbf{e}^{\prime}$. With this pinhole camera we want to image different regularly spaced objects of the same size into the image plane. Fig. 2-2 shows a side-view of this arrangement.


Fig. 2-2: Imaging with a pinhole camera
Each object point, e.g. the top of the objects-characterised by the arrows - will emit a bundle of rays. But only one single ray of this bundle will pass through the pinhole onto the image plane. Thus this ray indicates the top of the object in the image and hence its size. All other rays of this object point are of no interest for this image.
In this manner, if we construct the images of all the objects, we will see that the images will be smaller and smaller the more the distant the objects are from the pinhole. The pinhole is the center, where all imaging rays cross each other. Therefore we will call it the projection center.

To summarize: The farther away the object is located from the projection center, the smaller the image. This kind of imaging is called entocentric perspective. .

The characteristic feature of this arrangement is, that viewed in the direction that the light travels, we first have the objects, then the projection center and finally the image plane. The objects are situated some finite distance in front of the projection center.

But what are the reasons for the different impressions of spatial depth? With the wide angle lens, the distance to the first object y , is small whereas with the tele-lens this distance is considerably larger, by a factor of 6 . We shall simulate this situation now with the help of our pinhole-camera (fig. 2-3):


Fig. 2-3: Imaging with different object distances
The picture on top (case a) shows the same situation as in the preceding fig. 2-2. In the picture on bottom (case b) the distance to the first object is 3.5 times as large. In order to image the first object $y_{1}$ with the same size as in case a we have to expand the image distance by the same factor 3.5 correspondingly. We see now, that the three objects $y_{1}, y_{2}, y_{3}$ have nearly the same size, exactly as with the telephoto lens of fig. 2-1. Whereas in case a, the image of $y_{3}$ is roughly half the size of the image of $y_{1}$, it is in case $b$ three quarter of that size. There would have been no change in the ratio of the image sizes if the image distance e' in case $b$ would have been the same as in case a, only the total image size would become smaller and could be enlarged to the same size.
Conclusion: The only thing that is important for the ratio of the image sizes is the distance of the objects from the projection center.

To gain a clearer picture of the resulting spatial depth impression, we have to arrange the object scene in space, as shown in the following fig. 2-4:


Fig. 2-4: Positional arrangement of the objects in space

Here we have two rows of equally sized and equally spaced objects (arrows). The objects we may imagine to be the chess figures of fig. $2-1$. The pinhole camera is looking centrally and horizontally in between the two object rows at a distance el and height $h$ relative to the objects. The objects are marked with different colors for clarity. For the construction of the images taken with different object distances, $e_{1}$, we choose to view this
scene from above. This is shown in fig. 2-5:


Here the objects are represented by colored circles (view from above) and are situated on the gray background plane. The pinhole camera is shown by its projection center and its image plane. The image plane is perpendicular to the screen and is therefore only seen as a straight line. The projection rays (view from above) are colored according to the objects colors. Only one row of the objects is shown for the sake of clarity and not to overload the drawings. The two parallel straight lines, on which the objects are positioned intersect each other at infinity. Where is this point located in the image plane? It is given by the ray with the same direction as the two parallel straight lines passing through the projection center and intersecting the image plane.

If we tilt the image plane into the plane of the screen, we may now reconstruct all the images. The horizon line (skyline) is to be

Fig. 2-5: Constructing the images of the object scene
found at an image height h ' as the image of the height h of the pinhole camera above the ground. The point of intersection of the two parallel lines at infinity is located in the image plane on this horizon and is the far-point $\mathbf{F}$ for this direction. We transfer know all the intersection points of the projection rays with the perpendicular image plane onto the tilted image plane. The connecting line between the intersection point of the red projection ray and the far point gives the direction of convergence of the two parallel object lines in the image. In this way we may construct all the images of the different objects.

If we now rotate the image by 180 degrees and remove all auxiliary lines we then see very clearly the effect of different perspective for the two object distances. This is in rather good agreement with the photo of the introductory example fig. 2-1.

Imagine now, that we want to measure the heights of the different objects from their images. This proves to be impossible because equally sized objects have different image sizes depending on the object distance which is unknown in general. The requirement would be that the images should be mapped with a constant size ratio to the objects. Only in this case would equally sized objects result in equally sized images, independent of the object distance. In
order to see what this requirement implies, we will investigate the image size ratios of equally sized objects under the influence of the object distance (fig. 2-6):


Fig. 2-6: image size ratio and object distance
From fig. 2-6:

$$
\frac{y_{1}^{\prime}}{y_{1}}=\frac{e^{\prime}}{e_{1}} \quad \text { and } \quad \frac{\mathrm{y}_{2}^{\prime}}{\mathrm{y}_{2}}=\frac{e^{\prime}}{e_{2}}
$$

The image size ratio $y_{1}^{\prime} / y^{\prime}$, will be called $m$. Dividing the first by the second equation and knowing that $y_{1}=y_{2}$ will result in the following:

$$
\frac{y_{1}^{\prime}}{y_{2}^{\prime}}=m=\frac{e_{2}}{e_{1}}
$$

The image sizes of equally sized objects are inversely proportional to the corresponding object distances
Additionally with:

$$
t=e_{2}-e_{1}=y_{1}^{\prime} \cdot m-y_{1}^{\prime}
$$

we have:

$$
\begin{array}{|l|l|}
\hline e_{1}=\frac{t}{m-1} & e_{2}=\frac{t \cdot m}{m-1} \\
\hline
\end{array}
$$

If now $e_{1}$ becomes larger and larger, the ratio

$$
\frac{e_{2}}{e_{1}}=\frac{e_{1}+t}{e_{1}}
$$

approaches $I$, and hence also the image size ratio $y^{\prime} / y^{\prime}{ }_{2}$. In the limit, as $e_{1}$ reaches infinity, the image size ratio will be exactly I.

This is the case of telecentric perspective: the projection center is at infinity, and all equally sized objects have the same image size

Of course we may not realize this in practice with a pinhole camera, since the image distance $\mathrm{e}^{\prime}$ would also go to infinity. But with an optical trick we may transform the projection center to infinity without reducing the image sizes to zero. In front of the pinhole camera, we position a lens in such a way that the pinhole lies at the (image side) focal point of this lens. Details will be explained in chapters 3 and 4 (see fig. 2-7):


Fig. 2-7: object side telecentric perspective
The image of the pinhole imaged by the lens onto the left side in the object space is now situated at infinity. Only those rays parallel to the axis of symmetry of the pinhole camera will pass trough the pinhole, because it lies at the image side focal point of the lens. That's why all equally sized objects will be imaged with equal size into the image plane.
As a consequence of the imaging of the pinhole, we now have two projection centers, one on the object side (which lies at infinity) and one on the side of the images, which lies at a finite distance $e^{\prime}$ in front of the image plane. Therefore this arrangement is called object side telecentric perspective.

## Hypercentric perspective

If the pinhole is positioned with regard to the lens in such a way that its image (in object space on the left) is situated at a finite distance in front of the objects, the case of hypercentric perspective is realized (see fig. 2-8):


Fig. 2-8: hypercentric perspective
The projectied rays (passing through the pinhole) now have a direction, such that their backward prolongations pass through the object side projection center. All equally sized objects will now be imaged with larger size the farther they are away from the pinhole camera. Thus we may look "around the corner"!

At the end of this chapter we will have a look at some examples which are of interest for optical measurement techniques. The object is a plug-in with many equally sized pins. Fig.2-8 shows two pictures taken with a $2 / 3$ inch CCD-camera and two different lenses


Fig. 2-8: entocentric and telecentric perspective
The left side picture is taken with a lens with a focal length of 12 mm (this corresponds roughly to a standard lens of 50 mm focal length for 35 mm format $36 \times 24 \mathrm{~mm}$ ). Here we clearly see the perspective convergence of parallel lines, which is totally unsuitable for measurement purposes. On the other hand, the telecentric picture on the right, which is taken from the same position shows all the pins with equal size and is thus suited for measurement purposes.

The perspective convergence is not only true for horizontal views into the object space, but also for views from the top of the objects. This is shown in the following pictures for the same plug-in (fig. 2-9)


Fig. 2-9: Viewing direction form the top of the object
The picture with the standard lens ( $\mathrm{f}^{\prime}=12 \mathrm{~mm}$ ) shows only one pin exactly from top (it is the fourth counted from left). All others show the left or right side wall of the pins. For the picture with a three-times tele lens ( $\mathrm{f}^{\prime}=35 \mathrm{~mm}$ ), this effect is already diminished. But only the picture taken with a telecentric lens clearly shows all pins exactly from top. In this case, the measurement of all the pins is possible.

## 3. Gaussian Optics and its relation to central projection

### 3.1 The mathematical model of central projection

The linear camera model as used in digital image processing corresponds mathematically to a rotational symmetric projective transform. The points of an object space are related to the points of an image plane.
Geometrically we may construct this central projection by straight lines which proceed from the points in object space trough the projection center P to the intersection point with the projection plane (= image plane). These intersection points represent the image points related to the corresponding object points. (Fig. 1-1).


Fig. 3-1: The linear camera model
This model is hence represented by a projection center and the relative position of the image plane hereto. The position of the image plane is given by the distance c to the projection center 8camera constant) and by the intersection point of the normal from the projection center to the image plane (image principal point HP).

This normal represents an axis of rotational symmetry: all object points located in the same plane and with the same distance from this axis are imaged to the image plane with equal distance to the point HP.

Further on we may see that this transformation is a linear imaging of the object space onto the image plane. Straight lines in the object space are imaged onto straight lines in the image plane. However this imaging has no one to one correspondence. Each object point corresponds definitely to a single image point, but one single image point is related to an infinity of object points, namely all those points which lie on the projection ray through the projection center P.

## The projection rays are purely one-dimensional lines and the projection center is a mathematical dimensionless point.

A pinhole camera (camera obscura) realizes this model of an ideal imaging in approximation. Each image point is created as a projection of the pinhole from the object point. (geometrical optical shadow image). If we disregard for the moment the wave nature of light (diffraction) then this approximation will be better and better, the smaller we choose the dimension of the pinhole, and in the limiting case of vanishing pinhole diameter it represents the mathematical model.


Fig. 3-2: The pinhole camera as linear camera model

### 3.2 Central projection and Gaussian optics

How can we now approximate the linear camera model by a real imaging situation (with non vanishing aperture)?

The real physical imaging with light is connected with some energy transfer. In optics this is done with electromagnetic radiation in the optical waveband (UV to IR). Therefore this imaging is depending on the laws of propagation of electromagnetic waves, whereby the energy transport requires a finite aperture of the optical system. Because of the wave nature of this radiation, the image point corresponding to a certain object point is no longer a mathematical point, but has a certain finite extension.
For the spectral range of visible light the wavelength of the electromagnetic radiation is very small (approx. $0.5 \mu \mathrm{~m}$ ) resulting in extensions of the image points of the order of magnitude $5 \ldots 10 \mu \mathrm{~m}$. Therefore we may neglect in many cases (in a first step) the wave nature of light (later on we will again reconsider this).

It may be shown, that for the limiting case of vanishing wavelength the propagation of light can be described in form of light rays. These light rays represent the direction of energy flux of the electromagnetic waves and are normal to the wave front surfaces. The propagation of the light rays at refracting media (e.g. lenses) is described by the law of refraction (Fig. 3-3).


Fig. 3-3: The law of refraction

As we may see, the law of refraction is nonlinear, so we may not expect to get a linear transformation (central projection) by the imaging with light rays (Fig. 3-4).


Fig. 3-4: No point-like intersection of ray for wide aperture ray bundles
If we consider however only small angles between the rays and the normal to the lens surface, then we may approximate the sine-function by the angle alpha itself. In this case we come to a linear imaging situation. (at $7^{\circ}$ the difference between the sine function on the angle in rad is appr. 1 \%)

As may be shown, we have a one to one correspondence between object points and related image points for a system of centered spherical surfaces (e.g. 2 spherical surfaces $=1$ lens). The axis of symmetry is then the optical axis as the connecting line of the two centers of the spheres.

This new model of ray optics is to be understood as follows: From every point in object space there will emerge a homocentric ray bundle. Homocentric means that all rays have a common intersection point. This bundle of rays is transformed by the lens system (under the
assumption of small refracting angles) into a convergent homocentric ray bundle. The common intersection point of all rays in the image space defines the image point (Fig. 3-5).


Fig. 3-5: Common intersection points of the pencils of light for small refracting angles
The range of validity of this linear imaging is hence restricted to small refracting angles, and the region is given by a small tube-like area around the optical axis which is known as paraxial region.
The imaging laws in the paraxial region are described by Gaussian Optics.
As we see, Gaussian Optics represents the ideal situation of imaging and serves as a reference for the real physical imaging. All deviations from Gaussian Optics are considered as image errors (or aberrations $=$ deviations) although these are of course no errors in the sense of manufacturing tolerances but are due to the underlying physical lens. A huge a mount of effort during lens design is spent to the objective to enlarge the paraxial region so that the system may be used for practical applications. In this case the optic-designer is forced to use of course more then one single lens, whereby the expense and the number of lenses will increase with increasing requirements on image quality.

How does Gaussian Optic describe now this ideal imaging? Since all rays have a common intersection point, it will suffice to select such two rays for which we may easily find the direction of propagation. All other rays of the pencil will intersect at the same image point. The propagation of characteristic rays (and therefore the whole imaging situation) is given by the basis Gaussian quantities, also called cardinal elements (Fig. 3-6).


Fig. 3-6:Cardinal elements and the construction of images in Gaussian optics
Optical axis: straight line trough all centers of spherical surfaces
Focal points $\mathbf{F}, \mathbf{F}^{\prime}$ : Points of ray intersections for a pencil of rays parallel to the optical axis in object space (intersection point at $\mathrm{F}^{\prime}$ in image space) or in image space (emerging from $F$ in object space)

Principal planes H, $\mathbf{H}^{\prime}$ : Planes in object and image space, which are conjugate to each other and are imaged with a magnification ratio of +1 . Conjugate hereby means that one plane in the image of the other

Principal points $\mathbf{H}_{+}, \mathbf{H}^{\prime}$ : : The intersection points of the optical axis with the principal planes. For equal refractive indices in object and image space (e.g. air) we have: a ray with direction through the object side principal point $\mathrm{H}_{+}$will leave the optical system with the same direction and virtually emerges from the image side principal point $\mathrm{H}^{\prime}$.

Focal length $\mathbf{f}, \mathbf{f}$ ': directed distance HF (object side) or H'F' (image side). For systems in equal media (e.g. air) we have: $f=-f^{\prime}$.

With these cardinal elements we may construct the three rays (1) - (3) as in Fig. 3-6. Hereby it is of no importance if these rays pass really through the optical system or not, because all other rays will intersect at the same image point.

Ray 1: Ray parallel to the optical axis, will be refracted to pass through the image side focal point

Ray 2: Ray passing through the object side focal point, will be refracted to leave the optical system parallel to the optical axis

Ray 3: A ray passing through the object side principal point leaves the system parallel to itself and seems to originate from the image side principal point.
(Only valid for equal media in object and image space, e.g. air)

## Note:

The meaning of object and image space is as follows: Object space means: before the imaging through the optical system Image space means: after the imaging through the optical system From a purely geometrical viewpoint these two spaces may intersect each other. This will be the case e.g. for a virtuel image (cf. Fig. 3-8)

### 3.3 The finite apertures of ray bundles

In every technical realization of optical systems we have limitations of ray bundles e.g. at the edges of lenses, on tubes, stops etc. These limitations have effects on the cross sections of ray pencils and on the resulting image. In principle we exceed the range of validity of Gaussian Optics, the paraxial region. If we interpreted however Gaussian Optics as the ideal model of optical imaging, then we may find out the consequences of finite ray bundles on the imaging in a first approximation.

In general the ray pencils will be limited by a mechanical stop (the iris stop) within the optical system (without mechanical vignetting by other elements).
The limitations of the ray bundles in object- and image space will be found when we look at the image of this stop (i.e. by the approximation of Gaussian Optics) imaged back words into object space and forward into image space. The image of the stop in the object space is called entrance pupil (EP). You will see this entrance pupil when you look at the lens from the front side. Accordingly the image of the sop in image space is called exit pupil XP. This image may be seen, when looking at the lens from the rear side (Fig. 3-7).


Fig. 3-7: The limitations of ray pencils
The ray pencils are limited by the pupils (EP in object space and XP in image space).

If the object point is situated on the optical axis, the limiting rays of the pencil are called marginal rays RS. They proceed from the axial object point to the edges of the entrance pupil EP and from there to the edges of the exit pupil XP and finally to the axial image point (red).
For object points outside the optical axis the limiting rays of the pencil are called pharoid rays (PS). They proceed from the off-axis object point to the edges of the entrance pupil (EP) and from the edges of the exit pupil (XP) to the off-axis image point. (green)
The central ray of these pencils is called the principal ray PS (drown in blue). It proceeds from the object point to the center of the entrance pupil ( $\mathbf{P}$ ) and on the image side from the center of the exit pupil ( $\mathrm{P}^{\prime}$ ) to the image point.

The ratio of the diameters of exit- to entrance pupil is called the pupil magnification ratio $\beta_{\mathrm{p}}$.

$$
\beta_{P}=\frac{\Phi E X P}{\Phi E N P}
$$

The finite aperture of the ray pencils is characterized by the $\mathbf{f}$-number $\mathbf{K}$

$$
\mathrm{K}=\mathrm{f}^{\prime} / \varnothing \mathrm{EP}
$$

or, alternatively by the numerical aperture A. For systems surrounded by air we have:

$$
A=\sin \alpha_{\max }
$$

Hereby $\alpha_{\max }$ is half of the aperture angle of the ray pencil. Relation between f-number and numerical aperture:

$$
K=\frac{1}{2 \cdot A}
$$

The principal rays are the projection rays of the optical imaging. The intersection point of the principal ray with the image plane defines the focus of the image point, since it is the center of the circle of confusion of the ray pencil for image points in front or behind the image plane. Here we may notice an important difference with respect to mathematical central projection: caused by the aperture of the ray pencils, there is only a single object plane which is imaged sharply on the object plane. All points in front of behind this "focussing plane (EE)" are imaged as discs with a certain circle of confusion.

Because of the existence of a ray limiting aperture in object- as well as in image space (ENP in object space, EXP in image space) we now have two projection centers one at the center P of the entrance pupil on the object side and another one at the center $\mathrm{P}^{\prime}$ of the exit pupil on the image side. If the pupil magnification differs significantly from 1 then the object- and image side angles of the principal ray w and $\mathrm{w}^{\prime}$ will differ significantly.

The Gaussian imaging is described by imaging equations with reference to two coordinate systems, one on the object side (for all object side entities) with origin at the center P of the entrance pupil and another one on the image side (for all image side entities) with origin at the center $\mathrm{P}^{\prime}$ of the exit pupil.

The imaging equations are given by:

$$
e^{\prime}=f^{\prime} \cdot\left(\beta-\beta_{P}\right)
$$

$$
\begin{aligned}
& e=f^{\prime} \cdot\left(\frac{1}{\beta}-\frac{1}{\beta_{P}}\right) \\
& \frac{e^{\prime}}{e}=\beta \cdot \beta_{P} \\
& \frac{\tan \omega}{\tan \omega^{\prime}}=\beta_{P}
\end{aligned}
$$

with

$$
\begin{array}{lc}
f^{\prime}=H^{\prime} F^{\prime} & \text { focal length } \\
\beta_{P}=\frac{\Phi E X P}{\Phi E N P} & \text { pupil magnification ratio } \\
\beta=\frac{y^{\prime}}{y} \quad \text { magnification ratio=image size/object size }
\end{array}
$$

ФEXP and ФENP are the diameters of the exit and entrance pupils respectively.

## The sign conventions are taken according DIN 1335!

Some other forms of imaging equations are possible, if we take the origins of the coordinate systems in the focal point $\mathrm{F}, \mathrm{F}^{\prime}$. Then the object and image distances will be denoted by z and $z^{\prime}$.

$$
\begin{aligned}
& \beta=-\frac{z^{\prime}}{f^{\prime}}=-\frac{f}{z} \\
& z \cdot z^{\prime}=f \cdot f^{\prime}
\end{aligned}
$$

We will need this form later for telecentric systems.
The cardinal elements of Gaussian Optics with reference to the optical system are given by the lens manufacturer in form of "paraxial intersection lengths". These are the distances of the cardinal elements from the first lens vertex (for object side elements) and the distances of image side cardinal elements from the last lens vertex. These distances are denoted by " S " and "S'" with an index for the cardinal element, e.g. $\mathrm{S}_{\mathrm{F}} ; \mathrm{S}_{\mathrm{H}} ; \mathrm{S}_{\mathrm{EP}} / \mathrm{S}_{\mathrm{F}}^{\prime} ; \mathrm{S}^{\prime}{ }_{\mathrm{H}} ; \mathrm{S}^{\prime}{ }_{\mathrm{XP}}$. In order to explain the situation more clearly, we shall have a look onto a real optical system. It is a lens for a CCD-camera with C-mount (Xenoplan 1.4/17 mm from Schneider Kreuznach). The cardinal elements are given in the corresponding technical data sheet and are as follows:


- $\mathrm{f}^{\prime}=17.57 \mathrm{~mm}$
- $\mathrm{HH}^{\prime}=-3.16 \mathrm{~mm}$
- $\mathrm{K}=1.4$
- $\mathrm{S}_{\mathrm{EP}}=12.04 \mathrm{~mm}$
- $\mathrm{S}_{\mathrm{F}}=6.1 \mathrm{~mm}$
- $\mathrm{S}_{\mathrm{F}^{\prime}}=13.6 \mathrm{~mm}$
- $S^{\prime}{ }_{H^{\prime}}=-4.41 \mathrm{~mm}$
- $\mathrm{S}^{\prime}{ }_{\mathrm{AP}}=-38.91 \mathrm{~mm}$
- $\beta_{\mathrm{P}}=2.96$
- $d=24.93 \mathrm{~mm}$

Fig. 3-8 shows the ray pencils for this example.


Fig. 3-8: Apertures for the ray pencils of the lens $\mathbf{1 . 4 / 1 7} \mathbf{~ m m}$

Here the object- and image space intersect each other geometrically! The exit pupil XP for instance is situated in front of the system but belongs nevertheless to the image space, because it is the image of the aperture stop (iris) imaged through the optical system into the image space (virtuel image). Because of the relative large pupil magnification ratio ( $\beta_{\mathrm{P}} \sim 3$ !) the angles of the principal rays in object space $(\omega)$ and in image space $\left(\omega^{\prime}\right)$ differ significantly from each other.

### 3.4 The relation to the linear camera model

Now the question arises, how this model of Gaussian imaging (with two projection centers) is related to the mathematical model of central projection as outlined in Fig. 3-1 ?
In order to reconstruct the objects (out of the image coordinates) we have to reconstruct the object side principal ray angles w . This may not be done however with the help of $\mathrm{P}^{\prime}$ as projection center since the angles w and w ' are different in general.
We rather have to choose the projection center on the image side in such a way that we get the same angles to the image points, as one would have from the center of the entrance pupil $P$ to the corresponding object points.
We will denote this new projection center by $\mathrm{P}^{*}$, with its distance c to the image plane. Thus the following relation must hold:

$$
\omega=\omega_{P}
$$

$$
\text { or } \quad \tan \omega=\frac{y}{e}=\tan \omega_{P^{*}}=\frac{y^{\prime}}{c}
$$

Thus we have

$$
c=\frac{y^{\prime} \cdot e}{y}=\beta \cdot e
$$

This equation describes nothing else then the scaling of the projection distance on the image side:
Since the image has changed in size by the factor $\beta$ with respect to the object in the focussing plane, we also have to change the distance of the projection center (in object space $=e$ ) in image space correspondingly in order to have the same inclination angles of the projection rays.

Thus we have derived the connection to the mathematical central projection of Fig. 1-1: The common projection center is $\mathrm{P}=\mathrm{P}^{*}(\mathrm{P}=$ center of the entrance pupil) and its distance to the image plane is equal to the camera constant $\mathbf{c}$. For c we have:

$$
c=\beta \cdot e=f^{\prime} \cdot\left(1-\frac{\beta}{\beta_{P}}\right)
$$

(In photography, this distance is called the "perspective correct viewing distance). Fig. 3-9 shows this situation.


Fig. 3-9: The projection model and Gaussian Optics
In order to exclude all misinterpretations:
If one deals with error considerations with variation of imaging parameters (e.g. variation of the image plane position - as in chapter 5) we always must consider the real optical pencils. The projection ray in Fig. 3-9 on the image side is a pure fictions entity.

## Note concerning ray bundle limitations in Gaussian approximation

We may argue against the presented model of ray pencil limitations, that in general the pupil aberrations are large and that hence the Gaussian model may not be applied in reality. But indeed we only use the centers of the pupils to develop the projection model.

If we introduce canonical coordinates according to Hopkins [2] then we may treat - also in the presence of aberrations - the limitation of ray pencils and the projection model in a strictly analogous manner. The pupils are then represented by reference spheres and the wave front aberrations are expressed with reference to these spheres. The reduced pupil coordinates then describe the extensions of the ray pencils and are represented by unit circles - independent of the principal ray inclination angles. For a detailed treatment we refer to [2].

### 3.5 Consequences of the non vanishing aperture of the ray bundles I: depth of field and depth of focus

As a consequence of the finite aperture of the ray bundles there will be only one particular object plane (the focussing plan) which is imaged sharply onto the image plane. All other object points in front or behind this focussing plane are imaged as circles of confusion. The center of this circle of confusion is given by the intersection print of the principal ray with the image plane. The just tolerable diameter $\mathrm{d}_{\mathrm{G}}$ of this circle is depending on the resolution power of the detector.

In this way we have a depth of sharp imaging in the $T^{\prime}$ (depth of focus) in the image space giving just sufficiently sharp images. Conjugate to this there is a depth of field $\mathbf{T}$ in the object space object points which are within this region T may be imaged sufficiently sharp onto the image plane (see Fig. 3-10).


Fig. 3-10: Depth of field T and depth of focus $T^{\prime}$

Besides the permissable circle of confusion $\mathrm{d}^{\prime}$ the depth of field depends on the parameters of the optical imaging situation. One may describe the depth of field as a function of the object distance e from the entrance pupil EP or as a function of the magnification ratio $\beta$ : The sign conventions are valid according to the object- and image side coordinate system (e, y and $e^{\prime}, \mathrm{y}^{\prime}$ ) and are in correspondence with the German standard DIN 1335.

Depth of field as a function of the object distance
exact formula:

$$
T=\frac{2 \cdot f^{\prime 2} \cdot d_{G}^{\prime} \cdot K \cdot\left(e+\frac{f^{\prime}}{\beta_{P}}\right) \cdot e}{f^{\prime 4}-d_{G}^{\prime 2} \cdot K^{2} \cdot\left(e+\frac{f^{\prime}}{\beta_{P}}\right)^{2}}
$$

1. approximation:

$$
T=\frac{2 \cdot f^{\prime 2} \cdot d_{G}^{\prime} \cdot K \cdot e}{f^{\prime 4}-e^{2} \cdot K^{2} \cdot d_{G}^{\prime 2}}
$$

valid for:

$$
|e| \geq 10 \cdot \frac{f^{\prime}}{\beta_{P}}
$$

2. approximation:
$T \approx \frac{2 \cdot K \cdot d^{\prime}{ }_{G} \cdot e^{2}}{f^{\prime 2}}$ valid for:
$10 \cdot \frac{f^{\prime}}{\beta_{P}} \leq|e| \leq \frac{f^{\prime 2}}{3 \cdot K \cdot d_{G}^{\prime}}$

With:

$$
K=\frac{f^{\prime}}{\Phi E N P} \quad \mathbf{f}-\text { number }
$$

$\beta=\frac{y^{\prime}}{y} \quad$ magnification ratio (image/object)
$\beta_{P}=\frac{\Phi E X P}{\Phi E N P}$ pupil magnification ratio
${d^{\prime}}_{G}=$ permissable geometrical circle of confusion

Within the range of validity of the second approximation the depth of field T is inversely proportional to the square of the focal length (with constant distance e), i.e. reducing the focal length to one half will increase the depth of field by the factor 4.

Examples for the wide range of validity of the second approximation:
We choose $\mathrm{d}_{\mathrm{c}}=33 \mu \mathrm{~m} \quad \mathrm{k}=8 \quad \beta_{\mathrm{p}}=1$.
Then we have for

$$
\begin{array}{ll}
\mathrm{f}^{\prime}=100 \mathrm{~mm}: \quad 1</ \mathrm{e} /<12.5 \mathrm{~m} \\
\mathrm{f}^{\prime}=50 \mathrm{~mm}: \quad 0,5 \mathrm{~m}</ \mathrm{e} /<3 \mathrm{~m} \\
\mathrm{f}^{\prime}=35 \mathrm{~mm}: & 0,35 \mathrm{~m}</ \mathrm{e} /<1.5 \mathrm{~m}
\end{array}
$$

Depth of field as a function of magnification ratio $\beta$ :
exact formula:

$$
T=\frac{2 \cdot\left(1-\frac{\beta}{\beta_{P}}\right) \cdot K \cdot d_{G}^{\prime}}{\beta^{2}+\left(\frac{K \cdot d_{G}^{\prime}}{f^{\prime}}\right)^{2}}
$$

Approximation:

$$
T \approx \frac{2 \cdot K_{e} \cdot d_{G}^{\prime}}{\beta^{2}}
$$

With:

$$
\begin{aligned}
& K_{e}=K \cdot\left(1-\frac{\beta}{\beta_{P}}\right) \text { effective } \mathbf{f} \text { - number } \\
& |\beta| \geq 3 \cdot \frac{K \cdot d_{G}^{\prime}}{f^{\prime}}
\end{aligned}
$$

## Example for the validity range of this approximation:

We set: $\mathrm{d}^{\prime}{ }_{\mathrm{G}}=33 \mu \mathrm{~m} \mathrm{f}^{\prime}=50 \mathrm{~mm} \mathrm{~K}=16$
then we have:

$$
/ \beta />1 / 30
$$

Depth of focus:

$$
T^{\prime}=T \cdot \beta^{2}=2 \cdot K_{e} \cdot d_{G}^{\prime}
$$

### 3.6 Consequences of the finite aperture of ray bundles II: the wave nature of light diffraction effects

### 3.6.1 Diffraction limited (perfect) imaging

The light rays of geometrical optics represent only an approximate model for the description of the optical imaging.
If one examines structures (e.g. point images) with extension in the order of the wavelength of light $(0.5 \mu \mathrm{~m})$ then this model fails.
A perfectly corrected optical system - from the viewpoint of geometrical optics - will transform the diverging homocentric ray bundle of an object into a converging homocentric bundle, and the common intersection point of all the rays of this bundle is the image point $0^{\prime}$ (see Fig. 3-11).


Fig. 3-11: Homocentric ray bundle and spherical wave fronts
In spite of the results of geometrical optics (ray model) the image 0 ' is not an exact point but has a certain extension. The rays of the convergent bundle are only fictions entities, which represent nothing else then the normal to the wave front. For a perfectly corrected optical system * these wave fronts have the shape of spherical surfaces with the center in 0 '. They are limited by the diameter of the entrance pupil (EP on the object side) and by the diameter of the exit pupil (XP on the image side).
Because of these limitations of the spherical wave fronts, the resulting image is not a point without extension, but a blurred disk, the diffraction disc.
The form and extension of this disc depends on the wavelength of the light and on the form of the spherical wave front limitation.*

[^0]For circular apertures (as usual in optics) the distribution of the illumination in the image plane is given by the Airy diffraction disc. For points located on the optical axis this diffraction disc is rotational symmetric. Fig. 3-12 shows this Airy disc (for a wavelength of light of $0.5 \mu \mathrm{~m}$ and for effective $\mathrm{f}-\mathrm{No} \mathrm{K}_{\mathrm{e}}=11,16,32$, as well as an enlarged section.


Fig. 3-12: The Airy disc for on-axis object points
The extension of the airy disc depends on two parameters: the wavelength $\lambda$ of light and the aperture angle of the homocentric ray bundle.

Instead of on ideal point we have in the geometrical image plane a central disc surrounded by some diffraction rings (fringes) with rapidly decreasing intensity. The extension of the central disc up to the first dark ring is given by:

$$
r_{0}=1.22 \cdot \lambda \cdot K_{e} \quad K_{e}=\text { effective } \mathrm{f}-\text { number }=K \cdot\left(1-\frac{\beta}{\beta_{P}}\right)
$$

This is usually taken as the radius of the diffraction disc. Within this region we encounter approx. $85 \%$ of the total radiant flux, the first bright ring has $7 \%$ and the rest is up to the bright rings of higher order.

For object points outside the optical axis the homocentric ray bundle is no longer a rotational symmetric circular cone since the entrance pupil is viewed from the position of the object point as an ellipse. For larger field angles $\omega$ this effect will be even more pronounced. Therefore the Airy disc has now an elliptical shape, whereby the larger axis of the ellipse is pointing radially in the direction of the image center. For the larger and smaller semi-axis of the Airy disc (up to the first dark ring) we now have

$$
\begin{aligned}
d_{\omega t}^{\prime} & =\frac{2 \cdot r_{0}}{\cos ^{3} \omega} \quad \text { (large semi-axis) } \\
d_{\omega r}^{\prime} & =\frac{2 \cdot r_{0}}{\cos \omega} \quad(\text { small semi-axis })
\end{aligned}
$$

$\mathrm{r}_{0}=$ radius of the Airy disc on the optical axis
Besides this point spread function, the user is also interested in the image of an edge (edge spread function - especially for optical measurement techniques). This may be derived from the point spread function. Fig. 3-13 shows some diffraction limited edge spread functions (for the image center and for the effective f -numbers $\mathrm{K}_{\mathrm{e}}=11,16,32$ ).


Fig. 3-13: Diffraction limited edge spread functions for $K_{e}=11,16,32$

### 3.6.2 Diffraction and depth of focus

We may ask now, how the optical radiation is distributed outside the geometrical image plane and how the geometrical-optical cone of rays is modified.
This problem has been solved first by Lommel [5] and nearly at the same time by Struve. From this solution and with the properties of the so called Lommel-functions we may derive the following general conclusions:
(1) The intensity distribution is rotational symmetric to the optical axis.
(2) The intensity distribution in the neighborhood of the focus is symmetric to the geometrical image plane.

Lommel calculated the intensity distribution for various planes near the geometrical focus. From these data one may construct the lines of equal intensities (Isophotes).
Fig. 3-14 shows these lines in normalized coordinates $u$, $v$ (only for the upper right quadrant of the ray cone because of the mentioned symmetry properties).


Fig. 3-14: Isophotes of the diffraction intensity near focus
The normalized coordinates have been introduced in order to be independant of the special choice for the effective f-number and the wavelength of light.

For the normalized abscissa $u$ we have:

$$
u=\frac{\pi}{2 \cdot \lambda \cdot K_{e}^{2}} \cdot z
$$

Here z is the coordinate along the optical axis with origin in the Gaussian image plane.
For the normalized ordinate v we have:

$$
v=\frac{\pi}{\lambda \cdot K_{e}} \cdot r
$$

r is here the distance to the optical axis $\mathrm{r}=\sqrt{x^{2}+y^{2}}$

## Intensity distribution along the optical axis:

From the general Lommel relations one may calculate the intensity distribution along the optical axis (maxim intensity of the point spread function with defocussing).

The maximum intensity of the point spread function with defocussing (along the optical axis) is changing with a $\operatorname{sinc}^{2}$-function. The first point of zero intensity is given by

$$
\mathbf{u}=4 \pi
$$

in accordance with the isophotes of fig. 3-14.
Fig. 3-15 shows the intensity distribution along the optical axis.


Fig. 3-15: Intensity of the diffraction limited point spread function along the optical axis

The ratio of the maximum intensity of the point spread function with defocussing to the intensity in the Gaussian image plane is a special case of the so called Strehl intensity. It may by shown, that with pure defocussing a Strehl ratio of $80 \%$ corresponds to a wave front aberration of $\lambda / 4$ which is equivalent to the Rayleigh-criterion. It will be taken as permissable defocussing tolerance for diffraction limited systems.
The $\operatorname{sinc}^{2}$-function of fig. 3-15 decreases by $20 \%$ for $\mathrm{u} \sim 3.2 \sim \pi$. Thus we have for the diffraction limited depth of focus:

$$
T_{B}^{\prime}= \pm 2 \cdot \lambda \cdot K_{e}^{2}
$$

Here the question immediately arizes as to what will be the extension of the point spread function at these limits of the diffraction limited depth of focus. Here we need a suitable criterion for the extension of the point spread function.
For the Gaussian image plane we already defined it, it is the radius $r_{0}$ of the central disc. This disc contains $85 \%$ of the total radiant power. We generalize this criterion and define:
The extension of the point spread function in a defocussed image plane is given by the radius which covers $85 \%$ of the total radiant power.
With this we may generalize our question:

What are the limits of the point spread function as a function of defocussing and when do they coincide with the geometrical shadow border (defined by the homocentric ray pencil)?

The result of this (relatively complicated) calculation for $85 \%$ of encircled energy may be approximated in normalized coordinates u , v by a hyperbolic curve (Fig. 3-16).


Fig. 3-16: $85 \%$ point spread extension and the geometrical shadow region
We define: The limit of geometrical optic is given, if the $85 \%$ point spread diameter lies $10 \%$ over the geometrical shadow region. This is given according to fig. 3-16 for $\mathrm{a}= \pm 10$; $\mathrm{v}=11$

With that we may derive the following general statements:
(1) The extension of the diffraction disc is always larger (or in the limit equal) to the geometrical shadow limit
(2) With the above definition the geometrical shadow region is reached for $\mathrm{u} \sim 10$.
(3) For the extension of the diffraction disc in the focussing plane $(u=0)$, at the diffraction limited depth of focus ( $\mathrm{U}_{\mathrm{B}} \approx 3.2$ ) and at the approximation to the geometrical shadow region ( $u \approx 10$ ) we may thus deride the following table 1 when taking into consideration the relationships for the normalized coordinates u , v .

$$
r=\frac{\lambda \cdot K_{e}}{\pi} \cdot v \quad z=\frac{2 \cdot \lambda \cdot K_{e}^{2}}{\pi} \cdot u
$$

|  | u | z | $v$ | $r$ | $r r_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| focus | 0 | 0 | 3.83 | $\mathrm{r}_{0}=1.22 \lambda \cdot \mathrm{~K}_{\mathrm{e}}$ | 1 |
| diffr.limited depth | $\mathrm{u}_{\mathrm{B}} \simeq 3.2$ | $\mathrm{z}_{\mathrm{B}}=2 \lambda \cdot \mathrm{~K}_{\mathrm{e}}{ }^{2}$ | $\mathrm{v}_{\mathrm{B}} \cong 5.5$ | $\mathrm{r}_{\mathrm{B}} \cong 1.75 \cdot \lambda \cdot \mathrm{~K}_{\mathrm{e}}$ | $\cong 1.43$ |
| geometrical shadow limit | $\mathrm{u}_{\mathrm{S}}=10$ | $\mathrm{Z}_{\mathrm{S}}=6.4 \cdot \lambda \cdot \mathrm{~K}_{\mathrm{e}}{ }^{2}$ | $\mathrm{V}_{\mathrm{S}} \cong 11$ | $\mathrm{r}_{\mathrm{S}} \simeq 3.5 \cdot \lambda \cdot \mathrm{~K}_{\mathrm{e}}$ | $\cong 2.9$ |

Table 1: Diffraction limited disc extension for 3 different image planes

## Conclusions for depth of field considerations:

(1) Diffraction and depth of focus

As may be seen from fig. 3-16, it is not allowed to add the diffraction limited depth of focus to the geometrical depth of focus (defined by the permissable circle of confusion). One has to use only the geometrical depth of focus.
(2) Diffraction and permissable circle of confusion

In addition one has to examine if the permissable circle of confusion is exceeded by diffraction. Here the following procedure is useful:
The diameter of the diffraction disc approximates the geometrical shadow region for $\mathrm{u}=$ 10. Thus it follows (as has been shown):

$$
\mathrm{d}_{\mathrm{s}}^{\prime}=7 \cdot \lambda \cdot \mathrm{~K}_{\mathrm{e}} \quad\left(\mathrm{cf} \text { table, } \mathrm{d}_{\mathrm{s}}^{\prime}=2 \cdot \mathrm{r}_{\mathrm{s}}\right)
$$

This value is proportional to the effective f-number. It shall not be larger then the value of the permissable circle of confusion $\mathrm{d}^{\prime}$.

$$
\mathrm{d}_{\mathrm{G}}^{\prime} \geq \mathrm{d}_{\mathrm{s}}^{\prime}=7 \cdot \lambda \cdot \mathrm{~K}_{\mathrm{e}}
$$

From this we derive the maximum f-number at which we may calculate with the geometrical depth of focus:

$$
K_{e} \leq \frac{d_{G}^{\prime}}{7 \cdot \lambda} \quad \lambda=0.5 \mu \mathrm{~m}: \quad K_{e} \leq \frac{{d_{G}^{\prime}}_{G}[\mu m]}{3.5}
$$

The following table 2 shows the maximum effective f-number for various permissable circles of confusion $\mathrm{d}_{\mathrm{G}}$ up to which we may use the geometrical approximation

| $\mathrm{d}^{\prime}[\mu \mathrm{G}]$ | 150 | 100 | 75 | 60 | 50 | 33 | 25 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~K}_{\mathrm{e}}(\max )$ | 43 | 28 | 21 | 17 | 14 | 9.5 | 7 | 2.8 |

Table 2: maximum effective f-number for geometrical calculations
When the effective f-numbers are larger, then the diffraction limited disc dimensions will be larger then the permisssable circle of confusion.

In this case we have to give an estimate for the diameter of the diffraction disc at the limits of
the geometrical depth of focus. From the expression of the hyperbolic curve of fig. 3-16, one may derive after some calculation.

$$
d_{B}^{\prime} \approx \sqrt{\left(d_{G}^{\prime}\right)^{2}+2 \cdot K_{e}^{2}}
$$

With
$\mathrm{d}_{\mathrm{B}}=$ diameter of the diffraction disc at the limits of the geometrical depth of focus in $\mu \mathrm{m}$ $\mathrm{d}^{\prime}{ }_{\mathrm{G}}=$ geometrical circle of confusion (in $\mu \mathrm{m}$ )
$K_{e}=$ effective f-number $\left[K_{e}=K \cdot\left(1-\beta / \beta_{p}\right)\right]$

## Useful effective f-number

In the macro region together with small image formats we quickly exceed the limit for geometrical depth of focus validity which is given by

$$
K_{e} \leq \frac{d_{G}^{\prime}}{3.5}
$$

This is due to the fact that the permisssable circle of confusion $\mathrm{d}^{\prime}{ }_{\mathrm{G}}$ is very small (e.g. $33 \mu \mathrm{~m}$ for the format of $24 \times 36 \mathrm{~mm}^{2}$ ).
In this case it is advantageous to operate with the useful effective f-number $\mathbf{K}_{\mathbf{e F}}$.
This is the effective f-number where the disc dimension at the diffraction limited depth of focus is equal to the geometrical permissable circle of confusion.

$$
\begin{aligned}
& 2 \cdot r_{B}=3.8 \cdot \lambda \cdot K_{e}=d_{G}^{\prime} \\
& K_{e U}=\frac{d_{G}^{\prime}}{1.9} \quad(\lambda=0,5 \mu \mathrm{~m})
\end{aligned}
$$

Then the depth of focus is equal to the diffraction limited case.

$$
T_{B}^{\prime}= \pm 2 \cdot \lambda \cdot K_{e}^{2}
$$

## Example:

$$
\begin{array}{lll} 
& \mathrm{d}^{\prime}{ }_{\mathrm{G}}=33 \mu \mathrm{~m} & \mathrm{~K}_{\mathrm{eU}} \cong 18 \\
\text { we take: } & \beta_{\mathrm{P}} \cong 1 & \beta=-1(1: 1 \text { magnification }) \\
& K_{e}=\left(1-\frac{\beta}{\beta_{P}}\right)=2 \cdot K \\
& K=\frac{K_{e U}}{2} \approx 9 & T^{\prime}{ }_{B} \approx \pm 0.32 \mathrm{~mm}
\end{array}
$$

## 4. Telecentric perspective - projection center at infinity

### 4.1 Object side telecentric

### 4.1.1 Limitation of ray bundles

As has been shown in chapter 3.3, there are two projection centers with Gaussian optics, the center P of the entrance pupil in object space and the center $\mathrm{P}^{\prime}$ of the exit pupil in image space. Hence, for object side telecentric systems, the object side projection center P (and therefore the entrance pupil) has to be at infinity (c.f. chapter 2).
In the simplest case - which we choose for didactic reasons - this may be done by positioning the aperture stop at the image side focal point of the system. The aperture stop is then the exit pupil XP. The entrance pupil, EP, is the image of the aperture stop in object space, which is situated at infinity and is infinitely large. For the pupil magnification ratio we therefore have:

$$
\beta_{p}=\frac{D_{A P}}{D_{X P}}=0
$$

(see fig. 4-1).


Fig. 4-1: The principle of object side telecentric imaging
The principle rays (blue) are parallel to the optical axis (virtual from the direction of the center EP - dashed) passing through the edge of the object to the lens, from there to the center of XP and finally to the edge of the image. This principle ray is the same for all equally sized objects ( $\mathrm{y}_{1}, \mathrm{y}_{2}$ ) regardless of their axial position and it crosses the fixed image plane at one single point. For the object $\mathrm{y}_{2}$ it is (in a first approximation) the central ray of the diffusing circle*)

The result is, that the image size in a fixed image plane is independent of the object distance
*) Note: please compare the article: :"Bilateral telecentric lenses-a premise for high precision optical measurement techniques"

The marginal rays RS (red) and the pharoid rays PS (black and green) have an aperture angle alpha which depends on the diameter of the aperture stop AB (= XP). Here we may see the characteristic limitation for imaging with telecentric systems:

The diameter of the optical system (in the present case the lens) must be at least as large as the object size plus the size determined by the aperture angle of the beams.
The imaging equations become now:

$$
\begin{array}{ll}
e^{\prime}=-f^{\prime} \cdot \beta & \text { (counted from XP) } \\
e=\infty & \text { (counted from EP) } \\
\frac{\tan w}{\tan w^{\prime}}=\beta_{y}=0 \rightarrow \tan w=0 \\
c=f^{\prime} \cdot\left(1-\frac{\beta}{\beta_{y}}\right)=\infty & \text { ("camera constant") }
\end{array}
$$

### 4.1.2 Depth of field

The $f$-number $K=f^{\prime} / D_{E P}$ is now formally $=0$ and $1 / \beta_{p}$ approaches infinity. Therefore the usual expression for the depth of field will be undetermined. We may however transform the equation:

$$
\begin{gathered}
T=\frac{2 \cdot d^{\prime} \cdot K \cdot\left(1-\frac{\beta}{\beta_{p}}\right)}{\beta^{2}}=\frac{2 \cdot d^{\prime} \cdot f^{\prime} \cdot\left(\frac{1}{D_{B P}}-\frac{\beta}{D_{A P}}\right)}{\beta^{2}} \\
\lim _{D_{s 0 \rightarrow F}} T=\frac{-2 \cdot f^{\prime}}{D_{A P}} \cdot \frac{d^{\prime}}{\beta} \text { mit } \frac{f^{\prime}}{D_{A P} / 2}=\sin \alpha \text { folgt: } \\
T=-\frac{d^{\prime}}{\beta \cdot \sin \alpha}
\end{gathered}
$$

$\mathrm{A}=\sin (\mathrm{alpha})$ is the object side numerical aperture.
With the sin-condition

$$
\frac{\sin \alpha}{\sin \alpha^{\prime}}=\beta
$$

we have:

$$
T=\frac{d^{\prime}}{\beta^{2} \cdot \sin \alpha^{\prime}}
$$

$\mathrm{A}^{\prime}=\sin ($ alpha' $)=$ image side numerical aperture

### 4.1.3 Diffraction effects

The diameter of the diffraction disc $\mathrm{d}_{\mathrm{B}}$ is given by:

$$
\begin{gathered}
d_{B}^{\prime}=2,44 \cdot \lambda \cdot K_{e f t}=2,44 \cdot \lambda \cdot \frac{f^{\prime}}{D_{B P}} \cdot\left(\frac{\beta_{p}-\beta}{\beta_{p}}\right) \\
d_{B}^{\prime}=2,44 \cdot \lambda \cdot \frac{e^{\prime}}{D_{A P}} \\
d_{B}^{\prime}=1,22 \cdot \frac{\lambda}{\sin \alpha^{\prime}}=1,22 \cdot \frac{\lambda}{A^{\prime}}
\end{gathered}
$$

For the diffraction limited depth of focus $\mathrm{T}_{\mathrm{B}}^{\prime}$ and depth of field $\mathrm{T}_{\mathrm{B}}$ we have:

$$
\begin{gathered}
T_{B}^{\prime}=4 \cdot \lambda \cdot K_{e f f}^{2}=\left[\frac{2 \cdot f^{\prime}}{D_{z P}}\left(1-\frac{\beta}{\beta_{z}}\right)\right]^{2} \cdot \lambda \\
T_{B}^{\prime}=\left[\frac{2 \cdot e^{\prime}}{\beta_{y} \cdot D_{z P}}\right]^{2} \cdot \lambda \\
T_{B}^{\prime}=\frac{\lambda}{\sin ^{2} \alpha^{\prime}}=\frac{\lambda}{A^{\prime 2}} \\
T_{B}=\frac{1}{\beta^{2}} \cdot T_{B}^{t} \quad \frac{\sin \alpha^{\prime}}{\sin \alpha}=\frac{1}{\beta} \\
T_{B}=\frac{\lambda}{\sin ^{2} \alpha}=\frac{\lambda}{A^{2}}
\end{gathered}
$$

### 4.2 Bilateral telecentric

### 4.2.1 Afocal systems

For bilateral (object and image side) telecentric systems, the entrance pupil EP as well as the exit pupil XP have to be at infinity. This we may achieve only with a 2 -component system, since the aperture stop has to be imaged in object space as well as in image space at infinity. It follows that the image side focal point of the first component must coincede with the object side focal point of the second component. ( $\mathrm{F}_{1}^{\prime}=\mathrm{F}_{2}$ ).
Such systems are called afocal systems since the object- and image side focal points ( $\mathrm{F}_{\mathrm{G}}{ }_{\mathrm{G}}, \mathrm{F}_{\mathrm{G}}$ ) for the complete system are at infinity now. Fig. 4-2 shows the case of two components L1, L2 with positive power.


Fig. 4-2: Afocal systems
A ray parallel to the optical axis leaves the system also parallel to the optical axis, the intersection point lies at infinity, this is the image side focal point $\mathrm{F}_{\mathrm{G}}$ of the system. The same is true for the object side focal point $\mathrm{F}_{\mathrm{G}}$ of the system. (afocal means "without focal points") A typical example of an afocal system with two components of positive power is the Kepler telescope. Objects which practically lie at infinity will be imaged by this system again to infinity. The images are observed behind the second component with the relaxed eye (looking at infinity). Fig. 4-3 shows this for an object point at the edge of the object (e.g. the edge of the moon disk).


Fig. 4-3: The Kepler telescope
The clear aperture of the first component ( $\mathrm{L} 1=$ telescope lens) now acts as the entrance pupil $E P$ of the system. The image of EP, imaged by the second component ( $\mathrm{L} 2=$ eye-piece) is the exit pupil XP in image space (dashed black lines). The object point at infinity sends a parallel bundle of rays at the field angle w into the component L1. L1 produces the intermediate image $y^{\prime}$ at the image side focal plane $\mathrm{F}_{1}$. At the same time this is the object side focal plane for component L2. Hence this intermediate image is again imaged to infinity by the second
component L2 with a field angle of $\mathrm{w}^{\prime}$. The eye is positioned at the exit pupil XP and observes the image without focussing under the field angle $\mathrm{w}^{\prime}$.
Since the object as well as the image are situated at infinity (and are infinitely large), the magnification ratio is undetermined. In this case, for the apparent magnification of the image, it depends on how much the tangent of the viewing angle $w^{\prime}$ has been changed compared to the observation of the object without the telescope from the place of the entrance pupil (tangent of viewing angle w). In this way the telescope magnification is defined:

The telescope magnification is the ratio of the viewing angle ( $\tan w^{\prime}$ ) with the instrument to the viewing angle ( $\tan \mathbf{w}$ ) without the instrument.

$$
V_{F}=\frac{\text { tangent of the viewing angle w' with instrument }}{\text { tangent of the viewing angle w without instrument }}
$$

From fig. 4-3:

$$
\begin{aligned}
& \tan w=\frac{y^{\prime}}{f_{1}^{\prime}} \\
& \tan w^{\prime}=\frac{y^{\prime}}{f_{2}}=-\frac{y^{\prime}}{f_{2}^{\prime}}
\end{aligned}
$$

Hence the telescope magnification $\mathrm{V}_{\mathrm{F}}$ is given by:

$$
V_{F}=-\frac{f_{1}^{\prime}}{f_{2}^{\prime}}
$$

With a large focal length $f^{\prime}{ }_{1}$ and a short eye-piece focal length $f_{2}$ we will get a large magnification of the viewing angles. This is the main purpose of the Kepler-telescope.

### 4.2.2 Imaging with afocal systems at finite distances

With afocal systems one may however not only observe objects at infinity, but one may also create real images in a finite position. This is true in the case of contactless optical measurement technique and has been the main reason for choosing the Kepler-telescope as a starting point.
In order to generate real images with afocal systems we have to consider the following fact: we will get only real final images, when the intermediate image of the first component is situated in the region between the first component and the point $\mathrm{F}_{1}=\mathrm{F}_{2}$. Otherwise the second component would act as a magnifying lupe and the final image would be virtual. This is explained by fig. 4-4:


Fig. 4-4: real final images with afocal systems
The object is now situated between the object side focal point F1 and L1. Because of this, the intermediate image $y^{\prime}$ is virtual and is located in front of the object. This virtual intermediate image is then transformed by L2 into the real final image $y^{\prime \prime}$.
For the mathematical treatment of the imaging procedure, we now apply the Newton imaging equations one after the other to component L1 and L2.

## Intermediate image at component L1:

$$
\beta_{1}=-\frac{z_{1}^{\prime}}{f_{1}^{\prime}}=-\frac{f_{1}}{z_{1}} \quad \text { (Newton) }
$$

This intermediate image acts as the object for imaging by the second component L2 (in Fig. 44 the intermediate image is virtual, hence for the second component this is a virtual object!) With the relation:

$$
\mathrm{F}_{1}^{\prime}=\mathrm{F}_{2}(\text { afocal sytem })
$$

and thus:

$$
\mathrm{z}_{2}=\mathrm{z}_{1}^{\prime}
$$

Imaging at the second component (L2):

$$
\beta_{2}=-\frac{f_{2}}{z_{2}}=-\frac{z_{2}^{\prime}}{f_{2}^{\prime}} \quad \text { (Newton) }
$$

## Overall-magnification ratio:

$$
\begin{aligned}
& \beta_{G}=\beta_{1} \cdot \beta_{2} \\
& \beta_{G}=-\frac{z_{1}^{\prime}}{f_{1}^{\prime}} \cdot\left(-\frac{f_{2}}{z_{2}}\right)
\end{aligned}
$$

With $z^{\prime}{ }_{1}=z_{2}$ und $f_{2}=-f_{2}^{\prime}$ we have:

$$
\beta_{G}=-\frac{f_{2}^{\prime}}{f_{1}^{\prime}}
$$

with the telescope magnification $\mathrm{V}_{\mathrm{F}}$ this gives:

$$
\beta_{G}=\frac{1}{V_{F}}
$$

## The overall magnification ratio is independent of the object position (characterized by

 $z_{1}$ ) and constant.This is valid - in contrast to the object side telecentric imaging (where the image size has been constant only for a fixed image plane c.f. chapt. 4.1.1) -for all image planes conjugate to the different object planes. If the image plane is slightly tilted there is - to a first approximation no change in image size, because the image side principal ray is parallel to the optical axis. The image size error produced by the tilt angle is only dependent on the cosine of this angle (c.f. chapt. 5).

The image positions may be calculated with the help of the second form of the Newton equations:

$$
z_{1}=\frac{f_{1}^{\prime} \cdot f_{1}}{z_{1}^{\prime}} \quad z_{2}^{\prime}=\frac{f_{2}^{\prime} \cdot f_{2}}{z_{2}}
$$

Dividing the second by the first equation yields:

$$
\frac{z_{2}^{\prime}}{z_{1}}=\frac{f_{2}^{\prime} \cdot f_{2}}{f_{1}^{\prime} \cdot f_{1}} \cdot \frac{z_{1}^{\prime}}{z_{2}}
$$

and with $\mathrm{z}_{1}^{\prime}=\mathrm{z}_{2}$ we have:

$$
\begin{gathered}
z_{2}^{\prime}=\frac{f_{2}^{\prime} \cdot f_{2}}{f_{1}^{\prime} \cdot f_{1}} \cdot z_{1}=\left(-\beta_{G}\right)^{2} \cdot z_{1} \\
z_{2}^{\prime}={\beta_{G}}^{2} \cdot z_{1}
\end{gathered}
$$

For the axial magnification ratio alpha' we have:

$$
\alpha^{\prime}=\frac{d z_{2}^{\prime}}{d z_{1}}=\beta_{G}{ }^{2}=\text { const. }
$$

### 4.2.3 Limitation of ray bundles

We now position the aperture stop AB at $\mathrm{F}_{1}=\mathrm{F}_{2}$. The entrance pupil EP (as the image of the aperture stop in object space) is now situated at - infinity, and the exit pupil XP (as the image
of the aperture stop in final image space) is situated at + infinity. With that we may construct the ray bundle limitations as in fig. 4-5.


Fig. 4-5: limitation of ray bundles for a bilateral telecentric system
The principle rays (blue) are again the projection rays and represent the central ray of the imaging ray bundles. They start parallel to the optical axis (coming virtually from the direction of the center of EP-dashed) passing through the edges of the objects $y_{1}$ and $y_{2}$ up to component L1. From there through the center of the aperture stop (which is conjugate to EP , XP) to component L2 and finally proceeding parallel to the optical axis to the edge of the images and pointing virtually to the center of XP at infinity (dashed).
The marginal rays RS (red) have an aperture angle alpha which depends on the diameter of the aperture stop. They start at the axial object point (pointing backwards virtually to edge of EP ) up to L1, from there to the edge of the aperture stop up to component L2 and proceeding finally to the image (pointing virtually to the edge of XP at infinity).

The pharoid rays (pink and green) also have an aperture angle alpha. They are coming virtually from the edge of EP (dashed) to the edge of the objects, from there real up to component L1, then through the edge of the aperture stop up to L2 and finally to the edge of the image (pointing virtually to the edge of XP at infinity-dashed).

Here we may see again very clearly, that because of the bilateral telecentricity, equally sized object will give equally sized images, even for the different image planes.

## "Pupil magnification ratio $\boldsymbol{\beta}_{\mathrm{P}}$ "

Since the entrance pupil EP as well as XP are at infinity (and are infinitely large), we may not use the term magnification ratio but should speak correctly of the pupil magnification $\mathbf{V}_{\mathbf{P}}$. The principle rays of the Kepler-telescope (c.f. fig. 4-3) are now the marginal rays of the bilateral telecentric system, since the positions of pupils and objects/images have been interchanged. Therefore we may define the pupil magnification $\mathrm{V}_{\mathrm{P}}$ in an analogous way to the telescope magnification $\mathrm{V}_{\mathrm{F}}$ :

$$
\begin{gathered}
V_{p}=\frac{\text { tangent of marginal ray } \alpha^{\prime} \text { of XP }}{\text { tangent of marginal ray } \alpha \text { of EP }}=\frac{\tan w_{F}^{\prime}}{\tan w_{F}}=V_{F} \\
V_{P}=\frac{\tan \alpha^{\prime}}{\tan \alpha}=\frac{1 / 2 D_{A B}}{f_{2}} \cdot \frac{f_{1}^{\prime}}{1 / 2 D_{A B}}=-\frac{f_{1}^{\prime}}{f_{2}^{\prime}}=V_{F}
\end{gathered}
$$

### 4.2.4 Depth of field

The starting point is the formula which expresses the depth of field in terms of the magnification ratio $\beta$ (c.f. chapt. 3.5)

$$
T=\frac{2 \cdot d_{G}^{\prime}}{\beta^{2}} \cdot K_{e}
$$

We have:

$$
K_{e}=\frac{f^{\prime}}{D_{z P}} \cdot\left(1-\frac{\beta}{\beta_{y}}\right) \quad \frac{K_{e}}{\beta}=\frac{f^{\prime}}{D_{z P}} \cdot\left(\frac{1}{\beta}-\frac{1}{b_{y}}\right)=\frac{e}{D_{z P}}
$$

hence:

$$
T=\frac{2 \cdot d_{G}^{\prime}}{\beta} \cdot \frac{K_{e}}{\beta}=\frac{2 \cdot d_{G}^{\prime}}{\beta} \cdot \frac{e}{D_{B P}}=\frac{d_{G}^{\prime}}{\beta} \cdot \frac{2 e}{D_{B P}} \quad \text { mit } \quad \lim _{\varepsilon \rightarrow \infty} \frac{D_{B P}}{2 e}=\sin c
$$

This gives for the depth of field:

$$
T=\frac{d_{G}^{\prime}}{\beta \cdot \sin \alpha}
$$

## Depth of focus:

With

$$
\frac{\sin \alpha^{\prime}}{\sin \alpha}=\frac{1}{\beta} \quad \text { und } \quad T^{\prime}=\beta^{2} \cdot T
$$

we have:

$$
T^{\prime}=\frac{d_{G}^{\prime}}{\sin \alpha^{\prime}}
$$

## To top

### 4.2.5 Diffraction effects

Extension of the image disc:
Starting point is the normalized coordinate v from section 3.6.2 and solved for r :

$$
r=\frac{\lambda \cdot K_{e}}{\pi} \cdot v
$$

this gives:

$$
d^{\prime}=\frac{2 \lambda \cdot K_{e}}{\pi} \cdot v \quad \text { mit } \quad K_{e}=\frac{1}{2 \cdot \sin \alpha^{\prime}} \quad \text { folgt } \quad d^{\prime}=\frac{\lambda}{\pi \cdot \sin \alpha^{\prime}} \cdot v
$$

in the focal plane we have $\mathrm{v}_{0}=3.83$ :

$$
d_{0}^{\prime}=\frac{1,22 \cdot \lambda}{\sin \alpha^{\prime}}
$$

and at the edge of the diffraction limited depth of focus we have $\mathrm{v}_{\mathrm{B}}=5.5$

$$
d_{B}^{\prime}=\frac{1,75 \cdot \lambda}{\sin \alpha^{\prime}} \quad \frac{d_{B}^{\prime}}{d_{0}^{\prime}} \approx 1,43
$$

## Diffraction limited depth of focus

Starting point is the normalized coordinate $u$ from section 3.6.2 and solved for z :

$$
z=\frac{2 \cdot \lambda \cdot K_{e}^{2}}{\pi} \cdot u=\frac{\lambda \cdot u}{2 \cdot \pi \cdot \sin ^{2} \alpha^{\prime}}
$$

The depth of focus is twice this:

$$
T^{\prime}=\frac{\lambda}{\sin ^{2} \alpha^{\prime}} \cdot \frac{u}{\pi}
$$

The normalized coordinate $u$ for diffraction limited depth of focus is $u=3.2$ (Rayleighcriterion), hence:

$$
T^{\prime} \approx \frac{\lambda}{\sin ^{2} \alpha^{\prime}}
$$

and with

$$
\frac{\sin \alpha^{\prime}}{\sin \alpha}=\frac{1}{\beta} \quad \text { und } \quad T=\frac{1}{\beta^{2}} \cdot T^{t}
$$

we have for the depth of field:

$$
T=\frac{\lambda}{\sin ^{2} \alpha}
$$

## 5.Error budget for 2D optical measurement techniques

5.1 Projection center at finite distance (entocentric perspective)

### 5.1.1 The measurement principle

### 5.1.2 Determination of the magnification ratio

### 5.1.3 Parameters influencing the relative error in image size (delta $y^{\prime} / y^{\prime}$ )

- 5.1.3.1 The influence of edge detection
- 5.1.3.2 The influence of misaligned image plane
- 5.1.3.3 The influence of misaligned object plane
- 5.1.3.4 The influence of image plane tilt
- 5.1.3.5 The influence of object plane tilt
- 5.1.3.6 Deviation of the optical axis from the mechanical reference axis ("Bore sight")
- 5.1.3.7 Nonlinearity of the camera model (Distortion)
- 5.1.3.8 The overall relative error in the object


### 5.2 Projection center at infinity (telecentric perspective)

### 5.2.2 Bilateral telecentric

### 5.2.1 Object side telecentric

### 5.1 Projection center at finite distance (entocentric Perspective)

### 5.1.1 The measurement principle

In 2D Measurement techniques one generally calculates the object size from the image size and the (calibrated) magnification ratio. We first start with the linear camera modell (validity of Gaussian Optics). Nonlinear influences (distortion) will be treated in a later chapter. Then we have the fundamental relation for the object size $y$ :

$$
\begin{equation*}
\mathrm{y}=\frac{\mathrm{y}^{\prime}}{\beta} \tag{1}
\end{equation*}
$$

The uncertainties (errors) in the determination of the image size $y^{\prime}$ and the magnification ratio will introduce some errors in object size $y$. The relative measurement error in the object is given (to a first approximation) by the total differential of equ. (1):

$$
\begin{equation*}
\left|\frac{\Delta \mathrm{y}}{\mathrm{y}}\right|=\left|\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right|+\left|\frac{\Delta \beta}{\beta}\right| \tag{2}
\end{equation*}
$$

Hence the relative measurement error in the object is composed of two terms: the relative measurement error in the image and the error in the determination of the magnification ratio.

### 5.1.2 Calibration of the magnification ratio

This is usually done with a scale (divided rule) in the object plane. With the imaging parameters set to the nominal values for the application, we image the scale onto the image plane and determine the image size. For the moment we assume a distortionless optical system. The influence of distortion will be considered in chapter 5.1.3.7. Then the magnification ratio is given by the ratio of image size to object size:

$$
\beta_{0}=\frac{\mathrm{y}_{0}^{\prime}}{\mathrm{y}_{0}}
$$

The relative error is again given by the total differential:

$$
\begin{equation*}
\left|\frac{\Delta \beta_{0}}{\beta_{0}}\right| \leq\left|\frac{\Delta \mathrm{y}_{0}^{\prime}}{\mathrm{y}_{0}^{\prime}}\right|+\left|\frac{\Delta \mathrm{y}_{0}}{\mathrm{y}_{0}}\right| \tag{3}
\end{equation*}
$$

Here Delta $y^{\prime}{ }_{0}$ is the uncertainty of the image size which is influenced by the edge detection algorithm and the number of pixels of the image sensor(see section 5.1.3.1). The relative error in the object size for a glass scale is approximately:

$$
\frac{\Delta y_{0}}{y_{0}} \approx 5 \cdot 10^{-5} .
$$

### 5.1.3 Parameters influencing the relative error in image size (delta $y^{\prime} / y^{\prime}$ )

In the following sections we will have a closer look at the parameters which have an influence on the error of the relative image size in equ. (2). Many of these are connected with geometrical misalignment of the imaging geometry. The following figure gives an overview of these parameters.


Relative image size error:

$$
\frac{\Delta y^{\prime}}{y^{\prime}}=F\left[\Delta e, \Delta e^{\prime}, \Delta \eta, \Delta \kappa, \Delta \varepsilon\right]
$$

In addition there will be an error influence on the relative image size by the edge detection algorithm (ED) and by the distortion of the optical system (V)

$$
\frac{\Delta y^{\prime}}{y^{\prime}}=F[E D, V]
$$

### 5.1.3.1 Influence of edge detection

The image size is given by the edges of the object; they determine the image geometry. These edges have to be detected in the image. In order to have good accuracy, we have to use a pixel-synchronized image capture into the frame grabber. Then the edge is detected with the help of subpixel-edge detection algorithms with an accuracy in the sub-pixel range. A realizable value in practice is around $1 / 10$ of a pixel. The image size of the object to be measured is given by the number of pixels N . Then we have for the relative error in image size by edge detection:

$$
\begin{equation*}
\left|\frac{\Delta y^{\prime}}{y^{\prime}}\right|_{E D}=2 \cdot \frac{0.1(\text { pixel })}{N(\text { pixel })} \tag{4}
\end{equation*}
$$

The factor 2 is due to the fact that for the image size we have to detect two edges. Fig 1 shows the relative error for edge detection as a function of the number of pixels N .


Fig. 1: Relative error of image size by edge detection

### 5.1.3.2 Influence of shifted image plane

If the objects to be measured are located in different object planes then the sensor must be positioned accurately in the corresponding image plane, for which the magnification ratio has been calibrated. With a shift error delta e', we have an error in the image size, but no error in the magnification ratio since it has been taken from the calibration procedure. With the formula of chapt. 3 we get:

$$
\begin{equation*}
\left|\frac{\Delta y^{\prime}}{y^{\prime}}\right|_{B E}=\frac{\Delta e^{\prime}}{e^{\prime}}=\frac{\Delta e^{\prime}}{f^{\prime}\left(\beta_{p}-\beta\right)} \tag{6}
\end{equation*}
$$

With:

$$
\begin{gathered}
\mathrm{e}^{\prime}=\text { Distance exit pupil (XP) - image plane } \\
\mathrm{f}=\text { focal length of the lens } \\
B=\text { magnification ratio (negative }!) \\
\beta_{\mathrm{p}}=\text { pupil magnificatio ratio }
\end{gathered}
$$

Fig. 2 shows the influence of the image plane shift delta e' on the relative error as a function of the magnification ratio $\beta$ (delta e' = parameter) using a Schneider Kreuznach lens, 1:1.4/6 mm as an example ( $\mathrm{f}^{\prime}=12.65 \mathrm{~mm} \beta_{\mathrm{p}}=4.31$ ).


Fig. 2: Influence of image plane shift on the relative error

### 5.1.3.3 Influence of shifted object plane

In practical situations one wants to measure a lot of parts with the same geometry. In this case there will be slightly different object positions because of part tolerances and the mechanical transport mechanism. This will result in a slightly different magnification ratio compared to the calibrated value which leads to a different image size ( $y^{\prime}+$ delta $y^{\prime}$ ). Under the assumption of an unchanged nominal magnification ratio, this gives an error delta y in object size. From the formula of chapt. 3 we may derive:

$$
\begin{equation*}
\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{OE}}=\frac{\Delta \mathrm{y}}{\mathrm{y}}=\frac{\Delta \mathrm{e}}{\mathrm{e}}=\frac{\Delta \mathrm{e} \cdot \beta}{\mathrm{f}^{\prime}\left(1-\frac{\beta}{\beta_{\mathrm{p}}}\right)}=\frac{\Delta \mathrm{e} \cdot \beta}{\mathrm{c}} \tag{7}
\end{equation*}
$$

With:

> Delta $\mathrm{e}=$ Tolerance in object position
> $\mathrm{e}=$ Distance entrance pupil (EP) - object plane
> $\mathrm{f}^{\prime}=$ Focal length of the lens
> $\beta=$ Magnification ratio (negative $!)$
> $\beta_{\mathrm{p}}=$ Pupil-magnification ratio $\mathrm{c}=$ Camera constant

Fig. 3 shows the influence of object plane shift delta e on the relative error as a function of the magnification ratio $\beta$ (delta $\mathrm{e}=$ parameter) for the lens MCS 1:1.4/6 mm.


Fig. 3: Influence of object plane shift

### 5.1.3.4 The influence of image plane tilt

This effect results in an image height dependent shift of the image plane. Fig. 3a shows the geometrical relationships.


Fig. 3a: Geometrical relationships for tilted image plane
With the sin-theorem for the triangles ABC and ADE we get:

$$
\begin{aligned}
& \Delta y_{-}^{\prime}=-y^{\prime}\left[1-\frac{\cos w^{\prime}}{\cos \left(w^{\prime}-\kappa\right)}\right] \\
& \Delta y_{+}^{\prime}=+y^{\prime} \cdot\left[1-\frac{\cos w^{\prime}}{\cos \left(w^{\prime}+\kappa\right.}\right]
\end{aligned}
$$

This gives for the absolute values:

$$
\begin{aligned}
& \left|\frac{\Delta y^{\prime}}{\mathrm{y}^{\prime}}\right|=\left|1-\frac{\cos \mathrm{w}^{\prime}}{\cos \left(\mid \mathrm{w}^{\prime}-\kappa\right)}\right| \\
& \text { with } \cos (\alpha+\beta)=\cos \alpha \cos \beta-\sin \alpha \sin \beta \\
& \text { and } \sin \alpha=\frac{\tan \alpha^{z}}{\sqrt{1+\tan ^{2} \alpha}} \quad \cos \alpha=\frac{1}{\sqrt{1+\tan ^{2} \alpha}} \\
& \text { and } \quad \tan \mathrm{w}^{\prime}=\frac{\mathrm{y}^{\prime}}{\mathrm{e}^{\prime}} \quad \mathrm{e}^{\prime}=\mathrm{f}^{\prime}\left(\beta_{\mathrm{p}}-\beta\right)
\end{aligned}
$$

we have:

$$
\begin{equation*}
\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}}=1-\frac{1}{\cos \kappa+\sin \kappa \cdot \tan \mathrm{w}^{\prime}}=1-\frac{1}{\cos \kappa+\sin \kappa\left(\mathrm{y}^{\prime} / \mathrm{e}^{\prime}\right)} \tag{8}
\end{equation*}
$$

With good approximation this may be simplified:

$$
\begin{align*}
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}}=\frac{\cos \kappa+\sin \kappa \cdot \tan \mathrm{w}^{\prime}-1}{\cos \kappa+\sin \kappa \cdot \tan \mathrm{w}^{\prime}}=\frac{\tan \kappa \cdot \tan \mathrm{w}^{\prime}}{1+\tan \kappa \cdot \tan \mathrm{w}^{\prime}}} \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}} \approx \tan \kappa \cdot \tan \mathrm{w}^{\prime}=\tan \kappa \cdot\left(\mathrm{y}^{\prime} / \mathrm{e}^{\prime}\right)=\frac{\mathrm{y}^{\prime} \cdot \tan \kappa}{\mathrm{f}^{\prime}\left(\beta_{\mathrm{p}}-\beta\right)}} \tag{9}
\end{align*}
$$

$\kappa=$ Tilt angle of image plane
$\mathrm{w}^{\prime}=$ Image-side pupil field angle
$y^{\prime}=$ Image height
$\mathrm{e}^{\prime}=$ Distance exit pupil (XP) - image plane
Fig. 4 shows the influence of image plane tilt on the relative error for an image height of 10 mm as a function of the tilt angle in arc min, with the exact formula and with the approximation.


Fig. 4: Relative error by image plane tilt as a function of tilt angle ( $\mathbf{y}^{\prime}=\mathbf{1 0} \mathbf{m m}$ )
Fig. 5 shows the influence of image plane tilt on the relative error as a function of magnification ratio for two different tilt angles ( 3 and 6 arc min)


Fig. 5: Relative error by image plane tilt as a function of image height (Kappa $=\mathbf{3 0}$ arc min and 3 arc min )

### 5.1.3.5 The influence of object plane tilt

The influence of an object plane tilt relative to the mechanical refrence axis is equivalent to an object height dependent shift of the object plane.
With a tilt angle etha we have from equ. (7):

$$
\begin{align*}
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KOE}}=\frac{\Delta \mathrm{y}}{\mathrm{y}}=\frac{\Delta \mathrm{e}}{\mathrm{e}}=\frac{\mathrm{y} \cdot \tan \eta}{\mathrm{e}}=\frac{\mathrm{y}^{\prime} \cdot \tan \eta}{\beta \cdot \mathrm{e}} } \\
& \text { mit } \beta \cdot \mathrm{e}=\mathrm{f}^{\prime}\left(1-\frac{\beta}{\beta_{\mathrm{p}}}\right)=\mathrm{c}(\text { Kammerkonstante }) \\
\text { folgt: } & \quad\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KOE}}=\frac{\mathrm{y}^{\prime} \cdot \tan \eta}{\mathrm{f}^{\prime}\left(1-\beta / \beta_{\mathrm{p}}\right)}=\frac{\mathrm{y}^{\prime}}{\mathrm{c}} \cdot \tan \eta
\end{align*}
$$

Fig. 6 shows the effect of object plane tilt as a function of magnification ratio for an image height of $y^{\prime}=10 \mathrm{~mm}$. Here it is obvious that for larger magnification ratios the relative error becomes smaller. This is due to the fact that for smaller $\beta$ the extension of the object plane will be larger and hence the defocussing, Delta e, at the edge of the object field will increase.


Fig. 6: The influence of object plane tilt on the relative error

### 5.1.3.6 Deviation $\varepsilon$ of the optical axis from the mechanical reference axis ("Bore sight")

A tilt of the optical axis with respect to the mechanical reference axis has the same effect at the object and image planes, as shown by fig. (7)


Fig. 7: Effect of a tilt of the optical axis with respect to the mechanical reference axis
Therefore the relative error is composed of two parts according to equs. (9) and (10):

$$
\begin{gather*}
{\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}}=\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KOE}}} \\
{\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}}=\frac{\mathrm{y}^{\prime} \cdot \tan \varepsilon}{\mathrm{f}^{\prime}\left(\beta_{\mathrm{p}}-\beta\right)}+\frac{\mathrm{y}^{\prime} \tan \varepsilon}{\mathrm{f}^{\prime}\left(1-\frac{\beta}{\beta_{\mathrm{p}}}\right)}} \\
{\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}}=\frac{\mathrm{y}^{\prime} \tan \varepsilon\left(1+\beta_{\mathrm{p}}\right)}{\mathrm{f}^{\prime}\left(\beta_{\mathrm{p}}-\beta\right)}} \tag{11}
\end{gather*}
$$

Fig. 8 shows the cummulative influence on the relative error.


### 5.1.3.7 Nonlinearity of the camera model by distortion

The distortion is defined as the relative difference of the actual measurement point with respect to the linear case given by Gaussian Optics.

$$
\begin{aligned}
& \mathrm{V}=\frac{\mathrm{y}_{\mathrm{i}}^{\prime}-\mathrm{y}_{\mathrm{G}}^{\prime}}{\mathrm{y}_{\mathrm{G}}^{\prime}}=\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}_{\mathrm{G}}^{\prime}} \\
& \mathrm{y}_{\mathrm{i}}^{\prime}=\text { real image point coordinate } \\
& \mathrm{y}_{\mathrm{G}}^{\prime}=\text { Gaussian image point }
\end{aligned}
$$

$$
\begin{equation*}
\text { hence }\left[\frac{\Delta y^{\prime}}{y^{\prime}}\right]_{\mathrm{V}}=\mathrm{V} \tag{12}
\end{equation*}
$$

The theoretical distortion may be corrected via suitable algorithms. Then there remains additional distortion generated by manufacturing tolerances on the order of magnitude $5 \cdot 10^{-4}$ to $1 \cdot 10^{-3}$. With the help of suitable calibration targets this may be further reduced. This same calibration target may also be used for the determination of the magnification ratio (cf.section 5.1.2).

### 5.1.3.8 The total relative error in the object

According to the above treated relationships we now have for the total error:

$$
\begin{equation*}
\left[\frac{\Delta \mathrm{y}}{\mathrm{y}}\right]_{\mathrm{ges}}=\left[\frac{\Delta \mathrm{y}}{\mathrm{y}}\right]_{\mathrm{M}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{K}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BE}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{OE}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBF}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KOE}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{V}} \tag{13}
\end{equation*}
$$

If we look at these error terms for the reference lens (MCS 1:1.4/12) we see that the most dominant term is the object plane variation $\Delta \mathrm{e}$ for lenses with finite pupil positions and for large magnification ratios. Hence it makes no sense to use such lenses for magnification ratios $|\beta|>0.2$.
The following gives a an error budget for the total relative error for the magnification ratios $\beta$ $=-1 / 20$ and $\beta=-1 / 5 \quad(\operatorname{MCS} 1: 1,4 / 12 \mathrm{~mm})$
magnification ratio:

$$
\begin{aligned}
\beta= & -\frac{1}{20}: \\
& {\left[\frac{\Delta \mathrm{y}}{\mathrm{y}}\right]_{\mathrm{M}}=5 \cdot 10^{-5} \quad(\text { Glasmaßstab }) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{K}}=3 \cdot 10^{-4} \quad(1 \mathrm{kx} 1 \mathrm{k} \mathrm{CCD}) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BE}}=4 \cdot 10^{-5} \quad\left(\Delta \mathrm{e}^{\prime}= \pm 2 \mu \mathrm{~m}\right) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{OE}}=2 \cdot 10^{-4} \quad(\Delta \mathrm{e}=0,05 \mathrm{~mm}) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}}=1,5 \cdot 10^{-4} \quad(\kappa=3 \mathrm{bmin}) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KOE}}=7 \cdot 10^{-4} \quad(\kappa=3 \mathrm{bmin}) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}}=3 \cdot 10^{-4} \quad(\kappa=1 \mathrm{bmin}) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{V}}=5 \cdot 10^{-4} }
\end{aligned}
$$

This gives a maximum total error of:

$$
\left|\frac{\Delta y}{y}\right|_{\text {tot }} \approx 2,3 \cdot 10^{-3}
$$

Under the assumption of statistical independence of the various error terms we get the RMSvalue

$$
\left.\overline{\left[\frac{\Delta y}{y}\right.}\right]_{\text {tot }} \approx 1 \cdot 10^{-3}
$$

e.g. for an object of size 140 mm we obtain a mean error of approx. 0.15 mm .

## Magnification ratio $\boldsymbol{\beta}=-1 / 5$ :

The following error term will change:

$$
\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{OE}}=8 \cdot 10^{-4}(\Delta \mathrm{e}=0,05 \mathrm{~mm})
$$

This gives a maximum total error:

$$
\left[\frac{\Delta y}{y}\right]_{\text {tot }} \approx 3 \cdot 10^{-3}
$$

RMS-value:

$$
\left[\frac{\Delta y}{y}\right]_{\text {tot }} \approx 1,3 \cdot 10^{-3}
$$

### 5.2 Projection center at infinity (telecentric perspective)

### 5.2.1 Object side telecentric

Here the entrance pupil is at Infinity and is infinitely large (as the image of the aperture stop at the focal plane which itself is the exit pupil XP). Therefore we have for the pupil magnification ratio $\beta_{p}$

$$
\beta_{\mathrm{p}}=\frac{\varnothing \mathrm{AP}}{\varnothing \mathrm{EP}}=0
$$

From the general imaging equation (Coordinate origins in the pupils, cf. chapt. 3) we have:

$$
c=f^{\prime}!\left(1-\frac{\beta}{\beta_{p}}\right)=\infty \quad \text { "Camera constant" }
$$

(The projection center is at infinity)

$$
\begin{aligned}
& e^{\prime}=-f^{\prime} \beta \quad \text { (counts from EXP) } \\
& \mathrm{e}=\infty \quad \text { (counts from EP }) \\
& \frac{\tan \mathrm{w}_{\mathrm{p}}}{\operatorname{tanw}_{\mathrm{p}^{\prime}}^{\prime}}=\beta_{p}=0 \rightarrow \boldsymbol{\operatorname { t a n }} w_{p}=0
\end{aligned}
$$

## Error terms

We have:

$$
\begin{aligned}
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{OE}}=\frac{\Delta \mathrm{e} \cdot \beta}{\mathrm{c}}=0 \quad(\mathrm{c}=\infty) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KOE}}=\frac{\mathrm{y}^{\prime} \cdot \tan \eta}{\mathrm{c}}=0 \quad(\mathrm{c}=\infty) } \\
& {\left[\frac{\left[\Delta \mathrm{y}^{\prime}\right]}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}}=\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}}=\frac{\mathrm{y}^{\prime} \cdot \tan \varepsilon}{-\mathrm{f}^{\prime} \cdot \beta} \quad\left(\beta_{\mathrm{p}}=0\right) } \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BE}}=\frac{\Delta \mathrm{e}^{\prime}}{-\mathrm{f}^{\prime} \cdot \beta} \quad\left(\beta_{\mathrm{p}}=0\right) }
\end{aligned}
$$

All other error terms remain unchanged.
Thus we have for the relative total error of the object:

$$
\begin{equation*}
\left[\frac{\Delta \mathrm{y}}{\mathrm{y}}\right]_{\mathrm{ges}}=\left[\frac{\Delta \mathrm{y}}{\mathrm{y}}\right]_{\mathrm{M}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{K}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BE}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}}^{*}+\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{V}} \tag{14}
\end{equation*}
$$

Error estimation $1\left(\beta_{p}=0, f^{\prime}=200 \mathrm{~mm}, \boldsymbol{\beta}^{\mathbf{n}}=\mathbf{- 1} / 4\right)$ :
(All error tolerances as in 1.3.8)

$$
\begin{aligned}
& {\left[\frac{\Delta y^{\prime}}{y^{\prime}}\right]_{B E} \approx 4 \cdot 5^{-5} \quad\left(\Delta e^{\prime}=2 \mu m\right)} \\
& {\left[\frac{\Delta y^{\prime}}{y^{\prime}}\right]_{K B E} \approx 2 \cdot 10^{-4} \quad(\kappa=3 \mathbf{m i n})} \\
& {\left[\frac{\Delta y^{\prime}}{\dot{y}}\right]_{B S} \approx 6 \cdot 10^{-5} \quad(\kappa=1 \mathrm{~min})}
\end{aligned}
$$

This results in a maximum total error of

$$
\begin{aligned}
& {\left[\frac{\Delta y}{y}\right]_{t o t} \approx 1,1 \cdot 10^{-3}} \\
& {\left[\frac{\Delta y}{y}\right]_{t o t} \approx 6 \cdot 10^{-4}}
\end{aligned}
$$

Error estimation $2\left(\boldsymbol{\beta}_{\mathrm{p}}=\mathbf{0}, \mathrm{f}^{\prime}=100, \mathrm{\beta}^{\mathrm{s}}=\mathbf{- 1}\right.$ :

$$
\begin{aligned}
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BE}} \approx 2 \cdot 10^{-5} \quad\left(\Delta \mathrm{e}^{\prime}=2 \mu \mathrm{~m}\right)} \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{KBE}} \approx 1 \cdot 10^{-4} \quad(\kappa=3 \mathrm{~b} \min )} \\
& {\left[\frac{\Delta \mathrm{y}^{\prime}}{\mathrm{y}^{\prime}}\right]_{\mathrm{BS}} \approx 3 \cdot 10^{-5} \quad(\kappa=1 \mathrm{~b} \min )}
\end{aligned}
$$

Maximum relative total error:

$$
\left[\frac{\Delta y}{y}\right]_{t o t} \approx 1 \cdot 10^{-3}
$$

RMS-value:

$$
\overline{\left[\frac{\Delta y}{y}\right]_{t o t}} \approx 6 \cdot 10^{-4}
$$

### 5.2.2 Bilateral telecentric (object- and image side)

Both EP and XP are at infinity. Therefore (see ):

$$
\begin{aligned}
& f_{\text {tot }}^{\prime}=\infty \\
& \beta=-\frac{f_{2}^{\prime}}{f_{1}^{\prime}} \\
& \beta_{p}=-\frac{f_{1}^{\prime}}{f_{2}^{\prime}}=\frac{1}{\beta}
\end{aligned}
$$

We have

$$
\begin{equation*}
\left[\frac{\Delta y}{y}\right]_{t o t}=\left[\frac{\Delta y}{y}\right]_{M}+\left[\frac{\Delta y^{\prime}}{y^{\prime}}\right]_{K}+\left[\frac{\Delta y^{\prime}}{y^{\prime}}\right]_{V} \tag{15}
\end{equation*}
$$

## Error estimation:

Maximum rel. total error

$$
\left[\frac{\Delta y}{y}\right]_{t o t} \approx 8 \cdot 10^{-4}
$$

RMS-value:

$$
\left[\frac{\Delta y}{y}\right]_{t o t} \approx 6 \cdot 10^{-4}
$$

It becomes obvious here, that to reduce the relative error further we have to reduce the edge detection error and the residual distortion which are dominant now.

## 6. The bilateral telecentric lenses from Schneider-Kreuznach

Here we would like to present to you our bilateral telecentric lenses. There are 5 different types for different magnification ratios.


- Xenoplan 1:1/A'=0.14
- Xenoplan 1:2/ $\mathrm{A}^{\prime}=\mathbf{0} .14$
- Xenoplan 1:3/ $\mathrm{A}^{\prime}=0.14$
- Xenoplan 1:4/A'=0.13
- Xenoplan 1:5/A'=0.13


They have the following common characteristic properties:

- high image side numerial aperture of 0.14 resp. 0.13
- for conventional lenses this corresponds to an effective f-number of $K_{e}=3.5$ resp. 3.8
- bilateral telecentric
- they possess all the advantagesfor optical measurement techniques which have been explained in detail in this short course. (chapt. 4, chapt. 5)
- variable iris and excellent image quality at full f-number.
- at $A^{\prime}=0.09$ nearly diffraction limited
- large area of applicable object depth, which is far larger then the depth of field, e.g. for the lens $\mathbf{1 : 5 / 0 . 1 3}$ nearly 100 mm
- thats why all lenses have a focussing capability
- very low distortion
- e.g. 1:5/0.13 $\mathrm{V}_{\max }=2 \mu \mathrm{~m}$ absolute
- very good telecentricity - fractions of a micron

In what follows we shall explain the diagrams contained in the technical data sheets of these lenses since they are somewhat different from the usual presentation.

## 1.Gaussian data

In this block you will find the Gaussian optical data of the corresponding lens as explained in detail in this course (chap. 3, chap. 4). Sum d means the overall optical length of the system, counted from the first to the last lens vertex.

```
XENOPLAN 1:5/0.13
f' = 10541.mm 哣 = -21.546
sm = 52463.mm 法 = 51973. mm
s*': = 2064. mm EmAP = 225054.mm
HH* = 75350.mm \Sigmad = 258.5 mm
```


## 2. Telecentricity



This diagramm shows the deviations in $\mu \mathrm{m}$ from exact telecentricity as a function of normalized image height ( $y^{\prime}{ }_{\text {max }}=5.5 \mathrm{~mm}$ ) for a variation in object position of $+/-1 \mathrm{~mm}$. The parameters here are three different object to image distances (OO'). You will find the object side deviation as well as the image side deviation from telecentricity (in this example the latter is practically identical to zero.).

## 3. Distortion



Distortion is given as an absolute deviation (in $\mu \mathrm{m}$ ) as a function of normalized image height, and as before for three different object to image distances.

## 4. Spectral Transmission



The spectral transmission is given as a function of wavelength and is valid for a ray along the optical axis. It includes the absorption effects of the optical glass as well as the transmission losses by the optical anti-reflection coatings.

## 5. Modulation Transfer Function (MTF)



The MTF is given for three spatial frequencies ( $20,40,80 \mathrm{Lp} / \mathrm{mm}$ ) as a function of relative image height and for radial orientation (full lines) as well as for tangential orientation (broken lines) of the object features. The different diagrams in the data sheet show the MTF for different image side numerical apertures and for different object to image distances ( $\mathrm{OO}^{\prime}$ '). The spectral weighting function for which these diagrams are valid is given in the header of the data sheet. For an introduction to the meaning of the MTF cf: Optics for digital photography

## 6. edge spread function and edge width



There are three data sheets corresponding to edge spread functions. Each data sheet is valid for one single object to image distance ( $\mathrm{OO}^{\prime}$ ). The edge as an object is defined as a sharp step in luminosity jumping from 0 to $100 \%$ at a certain image height (Heavyside step function).The edge spread function gives the irradiance distribution in the image of this object. On the abscissa we have the extension of the edge spread function (1 box corresponds to $5 \mu \mathrm{~m}$, on the ordinate we have the relative irradiance in percent. The left column of these diagrams shows these function for full f-stop-nr and three different image heights allways for radial (full lines) and tangential (dashed lines) orientation of the edge. The right column represents the same situaion for a numerical aperture of 0.09 . On the bottom of the diagrams you may find the values for the edge widths for radial $\left(\mathrm{K}_{\mathrm{r}}\right)$ and tangential $\left(\mathrm{K}_{\mathrm{t}}\right)$ Orientation.

## Definition of the edge widths



The origin of the local coordinate system $u$ is positioned at the median ( $50 \%$-value) of the edge. Then the left edge width LK is defined as the width of the rectangle with equal area as the left side of the edge spread function (up to the origin) the right edge width RK is defined in analoguous way. The sum $\mathrm{LK}+\mathrm{RK}$ is the edge width K (for radial or tangential orientation of the edge).
It may be shown that with the above choice of the local coordinate system the edge width K will be a minimum.
Fuerthermore we then have a relative simple correspondance to the Optical Transfer Function (cf. [14]).

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(to chapt. 2)

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[^0]:    * Note: A perfect (or diffraction limited) optical system is given, if the wave front in the exit pupil departs less then $\lambda / 4$ from an ideal sphere (Rayleigh criterion).

